Richard Alan (Satterfield)
www.richard-alan.com .

Distinguishable Permutations

The term, Distinguishable Permutations, refers to a distinguishable subset of the permutations that comprise the sample space of a process. The distinguishable subset is defined by specifying the distinguishing characteristics that a member of sample space must have to be a member of the distinguishable subset. The example below is a simple coin toss process which illustrates the concept.

Example

A coin is tossed 8 times. What is the probability that the sequence of 8 tosses yields 3H and 5T ?

The 8 consecutive tosses is the process **P**. The first toss has two possible outcomes, H or T; for each of those possible outcomes the second toss has two possible outcomes, H or T. Thus, in 2 tosses there are 2x2 possible outcomes. And in 8 tosses, there are 2x2 ... x2 = 2^8 = 256 possible outcomes which is the sample space **S** of **P**. The distinguishing characteristics that a member of the sample space must have to be a distinguishable permutation is any combination of 3H's and 5T's. To ascertain the probability that the sequence of 8 tosses yields 3H's and 5T's, you simply count the number of ways 3H's could be arranged in 8 time slots (i.e., number of combinations of 8 things 3 at a time) and then divide by 256. Note that for every 8 tosses in which 3 of the tosses are H, the 5 tosses that were not H must be T. Thus, you of course must get the same answer finding the number of ways 5T's could be arranged in 8 time slots and then divide by 256. The following illustrates that fact:

Let n(3H) be the number of ways you could arrange 3 things in 8 slots. Then,

$n(3H) = {}_8C_3 = 8! / 3!(8-3)! = 8! / (3!\ 5!) = 56$

Let n(5H) be the number of ways you could arrange 5 things in 8 slots. Then,

$n(5T) = {}_8C_5 = 8! / 5!(8-5)! = 8! / (5!\ 3!) = 56$

Thus the probability of 3H, or of 5T, or of 3H and 5T == $P(3H \cap 5T) = 56/256$

It is ironic that the size of the distinguishable subset of permutations is the number of combinations which have the distinguishing mix of heads and tails. Thus, we used the combination formula in counting permutations.

Definition

Given n objects with

r of one type, and

s == n - r of another type

The number of distinguishable permutations of the objects is

${}_nP_r == n! / r!(n-r)! = n! / (r!\ s!)$