Richard Alan Satterfield 01/11/18
www.richard-alan.com rev. 11/11/18

Capacitance To Earth

Abstract

This paper presents the derivation of a circuit model of the electrical interaction between the charges in an elevated metal plate and the charges in earth, and it shows the interaction to be purely capacitive. This result was surprising since conductive paths in earth are generally in the megohms. The paper also presents experimental results showing plate/earth capacitance for various plate/earth configurations. The experiments revealed how capacitance varies with turf, plate height, plate orientation, and plate size. The experimental results were even more surprising.

Introduction

Consider a metal plate elevated a small distance above the earth. In accordance with the laws of physics there will be an interaction between each charge in the plate and each charge in the earth. Fig.1. is a circuit model of each interaction. The first circuit element in the model denotes the interaction between charge q1 in the plate with a different charge q1 in the earth; the second circuit element denotes interaction between charge q1 in the plate with charge q2 in the earth; etc. The element after the left ellipsis models interaction between charge q2 in the plate with charge q1 in earth, etc. The element after the right ellipsis models the last element under the assumption that there are N charges in the earth and M charges in the plate. The capacitor in each element simulates the non-conductive interaction of a plate charge and an earth charge; the resistor simulates the resistance in the conductive path within the earth to the earth charge. There is no resistor modeling of the conductive path within the plate since all such paths are near zero resistance. The ground symbol in each element denotes earth voltage potential at the location of the earth charge that the element pertains to; their use implies the assumption that the steady state voltage potential of earth is uniform everywhere. Some obvious limitations of the model are that inductance is not simulated and that propagation velocity of electrical phenomena is considered infinite.

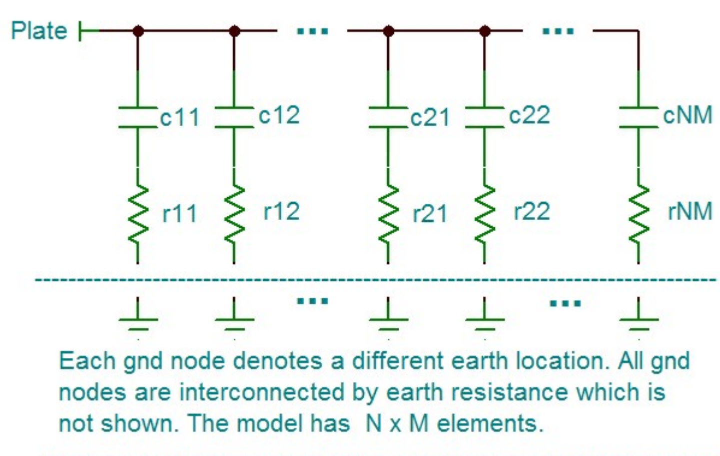


Figure 1. Circuit model of interaction between earth charges q1...qN and plate charges q1...qM. The model depicts N x M elements.

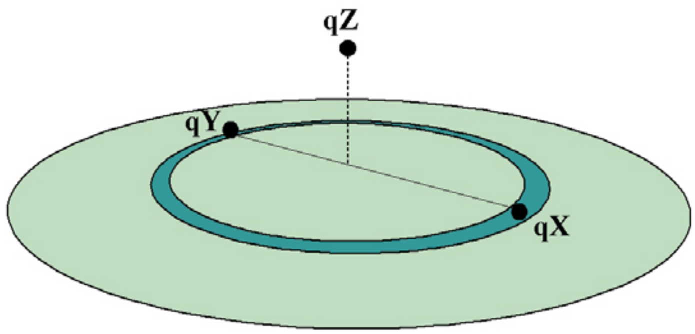
Some aspects of interaction between earth charges and plate charges are difficult to depict in a simple circuit model like that shown. For example, there is a complex network of resistive paths in earth between all ground nodes. This is noted in the Fig.1 narrative, but is not shown in the model.

Richard Alan Satterfield 01/11/18
www.richard-alan.com rev. 11/11/18

Equivalent Circuits

The complex circuit model in Fig.1 is useful for clarifying the physics of charge interaction between a metal plate and earth but its usefulness ends there because of the enormous number of elements. In this section a very simple circuit model is derived that is electrically equivalent to the complex model. As in the previous section, we once again consider a metal plate elevated a small distance above the earth. However in this case we begin by assuming there are only three charges involved. A single charge (qZ) in the plate and two charges (qX and qY) in earth. Also, we assume the earth charges are equidistant from the single plate charge as shown in Fig.2.

Figure 2. A single charge (qZ) in a metal plate elevated a small distance above earth is equidistant from two earth charges (qX and qY).



If we apply the circuit model in Fig.1 to the arrangement of the three charges shown in Fig.2, only two elements are required as shown in Fig.3a. Note that the resistive path between ground nodes is shown. This is not possible when many earth charges are involved because of the overwhelming complexity of the connections. Also note that since component values in an element are proportional to the distance between its earth charge and the plate charge, the X and Y component values must be equal and the elements must be identical. When two parallel circuit elements are identical their node voltages are identical. Thus, the nodes of such elements can be connected together as shown in Fig.3b without altering circuit behavior. In other words, the circuit models in Fig.3a and Fig.3b are absolutely equivalent because the circuit elements are identical. Therefore, by replacing the parallel components in Fig.3b with their single component equivalents, we have the simple circuit model shown in Fig.3c.

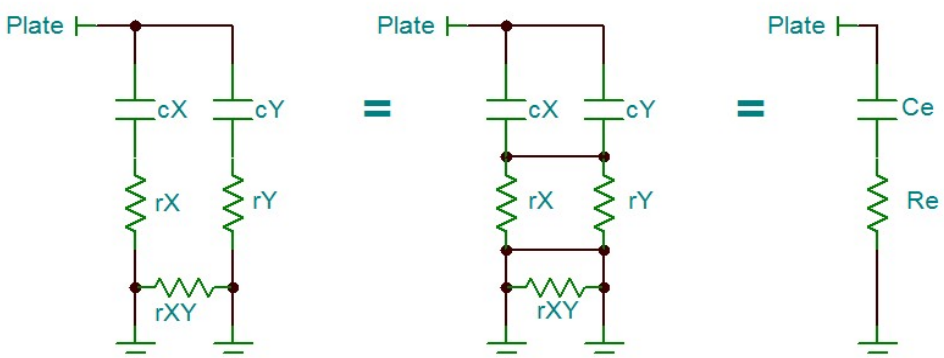


Figure 3. Circuit models of charge arrangement shown in Fig.2. Basis for each model: (a) derived from Fig.1 model, (b) derived by noting that X and Y component values identical, (c) derived by replacing parallel components with their single component equivalents.

The simple circuit model in Fig.3c applies to the two diametrically opposed earth charges shown in the dark green band in Fig.2. However, all of the other diametrically opposed earth charges that are not shown in the green band have exactly the same interaction with the plate charge as that modeled in Fig.3c. Thus, the circuit model of the interaction of the plate charge with all the earth charges in the dark green band would simply be the parallel combination of a multitude of elements like that shown in Fig.3c, one element per charge pair in the dark green band. The circuit model would consist of N circuit elements in parallel where in this case N represents the number of diametrically opposed charge pairs in the dark green band. This model is shown in Fig.4a. Since all of its elements are identical, the circuit model shown in Fig.4b would be absolutely equivalent. Therefore, by replacing the parallel components in Fig.4b with their single component equivalents, we have the simple circuit model shown in Fig.4c.

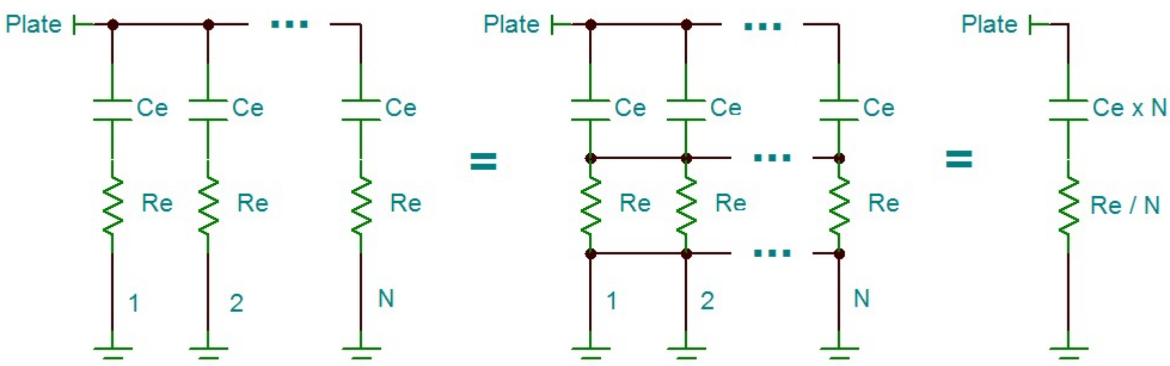


Figure 4. Circuit models of charge interaction between the charges in the dark green band of Fig.2. Basis for each model: (a) derived from Fig.3c model, (b) derived by noting that all elements are identical, (c) derived by replacing parallel components with their single component equivalents.

Note the values of the two components in the simple model of Fig.4c. The two component values are a function of N. No matter how you define the volume of the dark green band in Fig.2, N would be an astronomical number which would have a profound affect on the component values in the model. Although the value of Ce is obviously miniscule since it was based on the interaction of only three charges, it would be significant when multiplied by a large number like N. Equally important is the value of the conductive path in earth denoted by Re. Note that the resistance between metal stakes in turf is generally several megohms. This holds true for stake separation of one foot to at least several hundred feet. If the conductive path represented by Re were in the megohms, then the value of Re after being divided by N would essentially be zero. Thus the circuit model of a single charge in the metal plate interacting with the green band earth charges would be nothing more than a capacitor.

Obviously the approach used thus far in justifying a single capacitor as the circuit model for a charge in the metal plate and the charges in the green band, could be applied to other bands with the same result. Thus, the model for an extended area of the earth would simply be multiple capacitors in parallel, a single capacitor. Also, the same approach could be applied to other charges in the metal plate with the same result, a single capacitor. Thus, we adopt the following axiom.

@	The circuit model for an elevated metal plate and earth is a single capacitor.

Richard Alan Satterfield 01/11/18
www.richard-alan.com rev. 11/11/18

Setup For Measuring Plate/Earth Capacitance

The setup used for the measurement experiments presented in succeeding sections, is shown in Fig.6. The setup employs a sine wave generator to apply a signal to the metal plate whose plate/earth capacitance is to be measured. A scope is used to measure voltages VM1 and VM2. The scheme for determining capacitance involves adjusting Rp to achieve a certain phase relationship between the voltage VM1 and the current flow through Rp.

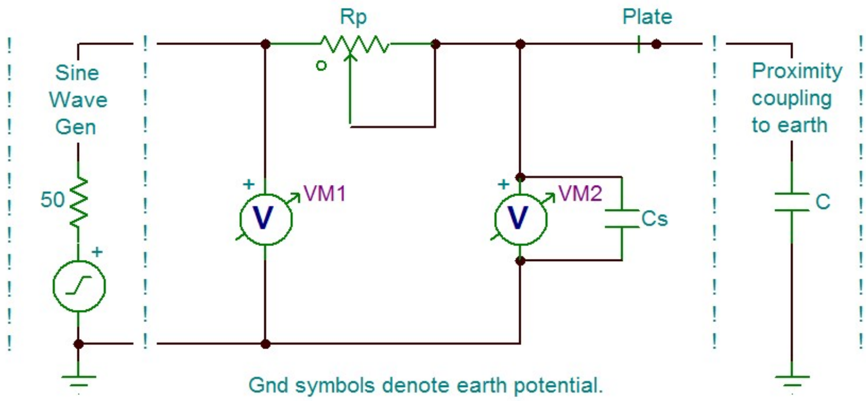


Figure 6. Test setup for measuring the proximity coupling parameters of a metal plate and earth.

The measurement procedure is simple. The voltage drop across Rp (VM1-VM2) is compared to the voltage applied to Rp (VM1). Since Rp is purely resistive the voltage drop across it is a voltage analog of the current through it. Thus, comparing the voltage drop with the voltage applied is tantamount to comparing the current through Rp with the voltage applied. The resistor Rp is adjusted until the phase difference between the two measurements (voltage and current analog) is 45 degrees. When that condition is achieved the resistance Rp in the circuit will be equal to the capacitive reactance of C+Cs where Cs denotes probe capacity in measuring VM2. You can express that relationship as shown in Eq.1, which defines C as shown in Eq.2. Fig.7 shows a typical scope display after Rp adjustment.

1.	$Rp = 1/(2\pi f\ (C+Cs))$	Rp adjusted to equal total capacitive reactance
2.	$C = 1/(2\pi f\ Rp) - 20p$	Solve Eq.1 for C and set Cs to the x10 probe capacitance of 20 pf

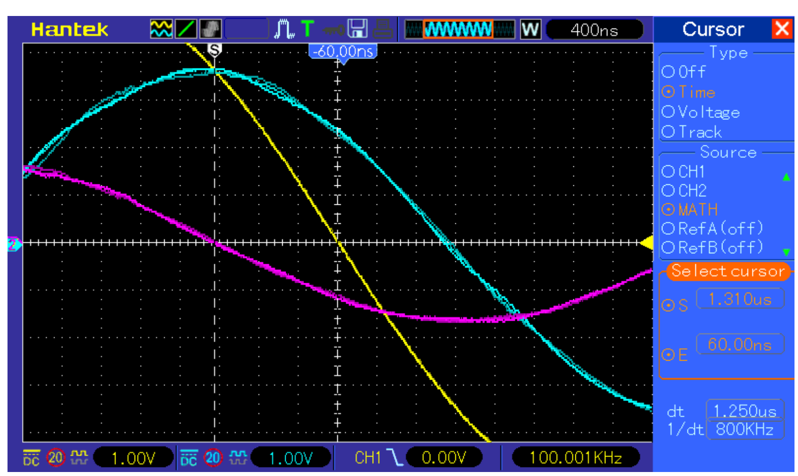


Figure 7. Scope display of VM1 (yel), Vm2 (grn), and VM1-VM2 (red). In this display the sine wave frequency has been set to 100 kHz. Therefore a phase shift of 45 degrees would correspond to a time shift of 1.25 us. Note that the current analog (red) leads the voltage applied to Rp (yel) by 45 degrees, which is a well known relationship for RC networks..

Plate/Earth Experiment #1

The plate/earth Configuration is a vertically oriented 24" x 12" metal plate elevated so its center is 19" above the earth surface. The earth's surface is concrete with imbedded rebar.

The calculated proximity coupling based on the measurements shown below are :

C1 = 25.4 p
C2 = 23.0 p

1023

f1 = 50 kHz
Rp_1 = 71.7 k

f2 = 100 kHz
Rp_2 = 37.0 k

Plate/Earth Experiment #2

This experiment is the same as experiment #1 except that the earth surface is turf rather than concrete. The plate/earth Configuration is a vertically oriented 24" x 12" metal plate elevated so its center is 19" above the earth surface.

The calculated proximity coupling based on the measurements shown below are :

C1 = 22.4 p
C2 = 21.8 p

1027

f1 = 50 kHz
Rp_1 = 75.0 k

f2 = 100 kHz
Rp_2 = 38.1 k

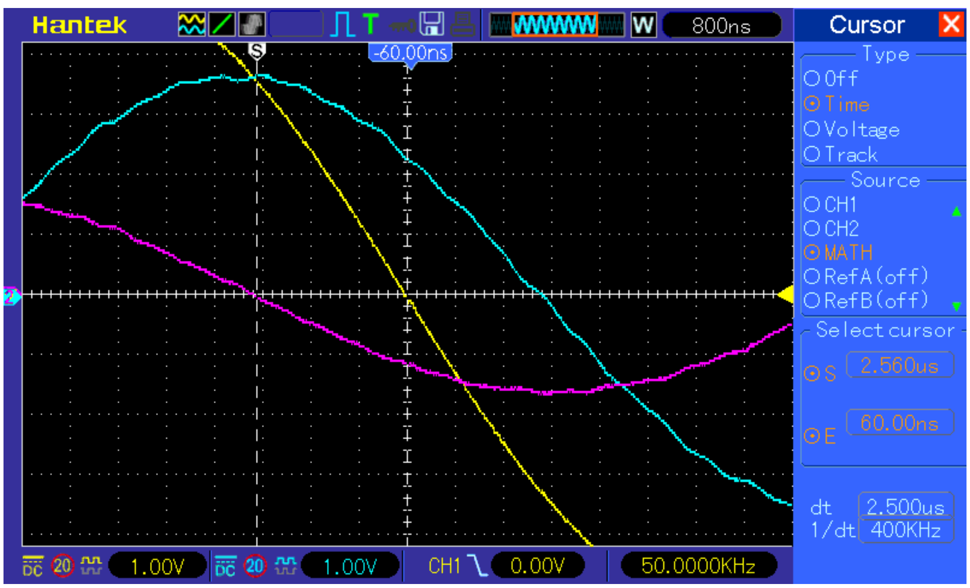
Plate/Earth Experiment #3

This experiment is the same as experiment #2 except that the plate is at a greater height. The plate/earth Configuration is a vertically oriented 24" x 12" metal plate elevated so its center is 50" above the earth surface.

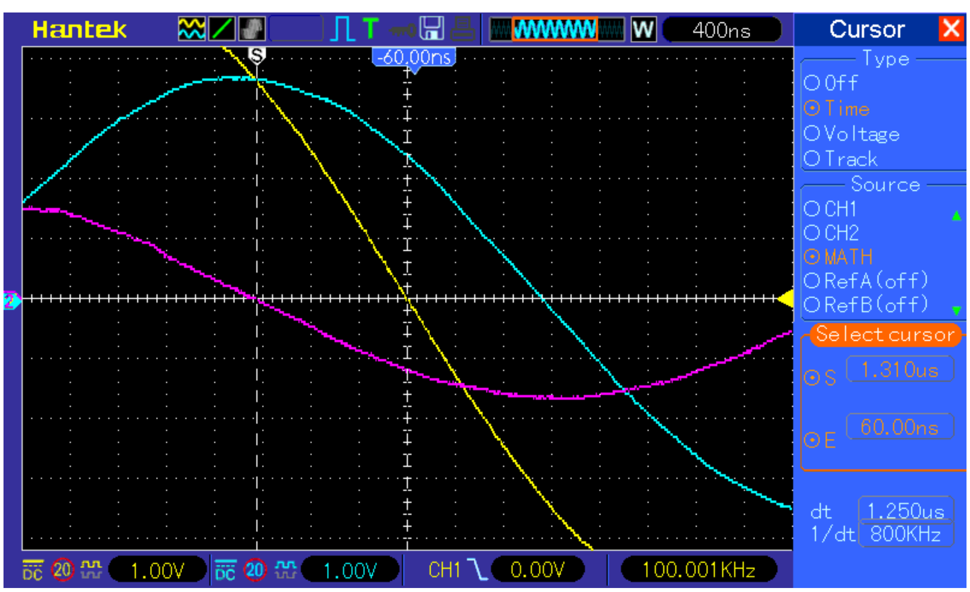
The calculated proximity coupling based on the measurements shown below are :

C1 = 22.4 p
C2 = 22.0 p

1027



f1 = 50 kHz
Rp_1 = 75.0 k



f2 = 100 kHz
Rp_2 = 37.9 k

Plate/Earth Experiment #4

This experiment is the same as experiment #2 except that the plate size is doubled. The plate/earth Configuration is a vertically oriented 24" x 24" metal plate elevated so its center is 19" above the earth surface.

The calculated proximity coupling based on the measurements shown below are :

C1 = 41.6 p
C2 = 38.1 p

1111

f1 = 50 kHz
Rp_1 = 51.7 k

f2 = 100 kHz
Rp_2 = 27.4 k

Richard Alan Satterfield 01/11/18
www.richard-alan.com rev. 11/11/18

Plate/Earth Experiment #5

This experiment is the same as experiment #4 except that the plate is at a greater height. The plate/earth Configuration is a vertically oriented 24" x 24" metal plate elevated so its center is 50" above the earth surface.

The calculated proximity coupling based on the measurements shown below are :

C1 = 33.4 p
C2 = 29.3 p

1111

f1 = 50 kHz
Rp_1 = 59.6 k

f2 = 100 kHz
Rp_2 = 32.3 k

Richard Alan Satterfield 01/11/18
www.richard-alan.com rev. 11/11/18

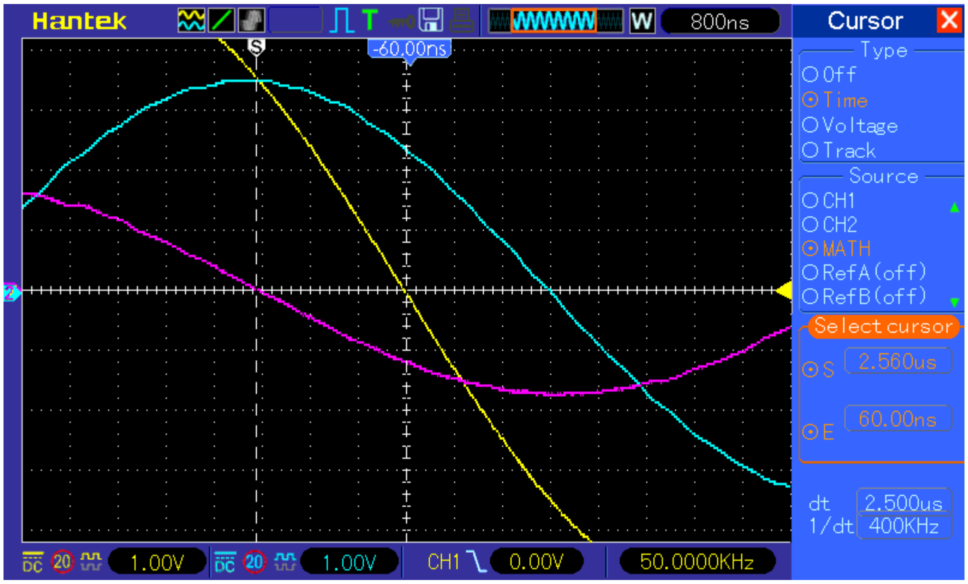
Plate/Earth Experiment #6

This experiment is the same as experiment #2 except that the plate is oriented horizontally instead of vertically. The plate/earth Configuration is a horizontally oriented 24" x 12" metal plate elevated so its center is 19" above the earth surface.

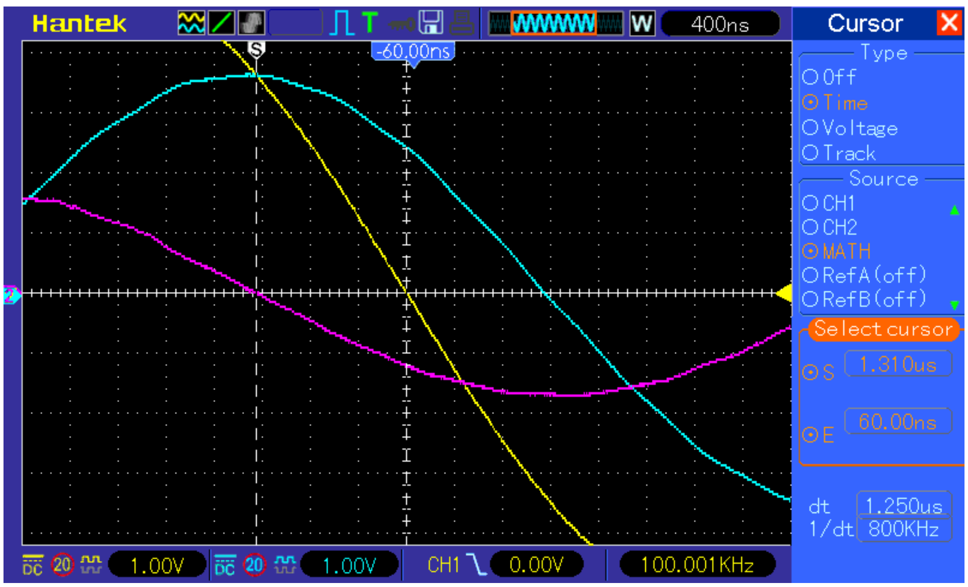
The calculated proximity coupling based on the measurements shown below are :

C1 = 37.0 p
C2 = 34.7 p

1111



f1 = 50 kHz
Rp_1 = 55.8 k



f2 = 100 kHz
Rp_2 = 29.1 k

Interpretation of Experimental Results

Capacity vs Earth Surface (Exp.#1 vs Exp.#2)

Capacitance was measured for two types of earth surface, turf and concrete with rebar. Surprisingly, the concrete with rebar only showed a 6 percent increase in capacitance over turf.

Capacity vs Plate Elevation (Exp.#2 vs Exp.#3) and (Exp.#4 vs Exp.#5)

Capacitance vs elevation was measured for two different plate sizes. The two elevations tested were 19" and 50"; higher elevation 260 percent greater than lower elevation. Surprisingly, the elevation change made virtually no difference in the capacity measurement using the 24" x 12" plate. Even with the larger 24" x 24" plate the reduction in capacitance at the higher elevation was only about 20 percent.

Capacity vs Plate Orientation (Exp.#2 vs Exp.#6)

Capacitance vs orientation was measured for the smaller 24" x 12" plate size. Two orientations were tested, vertical and horizontal. The horizontal orientation showed an almost 60 percent increase in capacitance. This result is surprising since changing plate orientation simply increases the elevation of half the charges in the plate, and decreases the elevation of the other half. You would expect the affect of changing plate orientation to either be zero or be similar to changing plate elevation. But, for some reason the effect is much more profound.

Capacity vs Plate Size (Exp.2 vs Exp.4) and (Exp.3 vs Exp.5)

Two different plate sizes were used, 24" x 12" and 24" x 24". Since the large plate is 100 percent larger than the small plate, the capacitance of the large plate was expected to be 100 percent greater than the small plate. The experiment was conducted at two different elevations, 19" and 50". At the lower elevation the measured increase in capacitance with larger plate size was almost 80 percent which was less than expected but not significantly different from what was expected. However, at the higher elevation the measured increase in capacitance with larger plate size was only 40 percent. This was surprising because at any elevation you would have expected the increase to be proportional to the increase in plate size.

Capacity vs Frequency (Exp.1 ... Exp.6)

Two capacity measurements were made for each experiment, one measurement at 50 KHz and one at 100 KHz. The lower frequency measurements were all from 2 to 9 percent higher than the higher frequency measurements.