Richard Alan (Satterfield)
www.richard-alan.com .

<u>Summary of PSU Lesson 3</u>

<u>Multiplication Principle and Replaceable Sets</u>

- If there are **m** ways of doing a first thing, and for each of the first things there are **n** ways of doing a second thing, then the Multiplication Principle tells us there are **m × n** ways of doing both things. Furthermore, this same approach may be applied to three things, or four things, or any number of things.

  Example: There are 2 ways of tossing a coin the first time, and for each of those first 2 tosses there are 2 ways of tossing the coin the second time, then the Multiplication Principle tells us there are 2x2 ways of tossing a coin 2 times. Furthermore, there are 2x2x2 ways of tossing a coin 3 times, etc.

- If you have **n** things that **<u>are</u>** replaceable, and that are distinguishable from one another, then the Multiplication Principle tells us that if order matters the maximum number of possible arrangements **m** of the **n** things without repeatition is given by:

  $$\mathbf{m = n*n* \ldots *n = n^n}$$

  Example: You have 3 distinguishable things **a b c**, that are replaceable, then the possible arrangements **m** of the 3 things are **aaa aab aac, aba abb abc, aca acb acc, baa bab bac, bba bbb bbc, bca bcb bcc, caa cab cac, cba cbb cbc, cca ccb ccc,** which is given by

  $$\mathbf{m = n^n} = 3^3 = 3*3*3 = 27$$

<u>Permutations and Non-Replaceable Sets</u>

- If you have **n** things that are **<u>not</u>** replaceable, and that are distinguishable from one another, then ascertaining the number of possible arrangements **m** of the **n** things (which is called permuting **n** things **n** at a time) is given by:

  $$\mathbf{m = n!}$$

- If you have **n** things that are **<u>not</u>** replaceable, and that are distinguishable from one another, and you create a subset of size **a** by selecting from the **n** things, then ascertaining the number of possible arrangements **m** of the **a** things (which is called permuting **n** things **a** at a time) is given by:

  $$\mathbf{m = n! / (n-a)!}$$

  Example: You have 7 things that are **<u>not</u>** replaceable; and that are **<u>distinguishable</u>** from one another, and you create a subset **a** of size 4 by selecting from the 7 things, then the number of possible arrangements **m** of the subset **a** is given by:

  $$\mathbf{m = n! / (n-a)!} = 7! / (7-4)! = 7*6*5*4 = 840$$

- If you have **n** things that are **<u>not</u>** replaceable; and which are comprised of 3 subsets of things **a**, **b**, and **c**; and where the members within each of the 3 subsets are indistinguishable from one another; then there are **m** = **n! / (a! b! c!)** discernable arrangements (called ***distinguishable permutations***) of the **n** things. Note that the algorithm for **m** can be extended to include any number of subsets.

  Example: If you have 7 things that are **<u>not</u>** replaceable; and which are comprised of 2 subsets of things **a** and **b**; and where the number of members in **a** is 4; and the number of members in **b** is 3; and where the members within each of the 2 subsets are **<u>indistinguishable</u>** from one another; then the **<u>distinguishable</u>** **<u>permutations</u>** of the 7 things are:

  m = **n! / (a! b!)** = 7! / (4! 3!) = 7*6*5 / 3*2*1 = 35