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The Wave Equation

Abstract

This note presents the derivation of the general wave equation for a transverse wave propagating in the positive X direction in an XYZ coordinate system.

Derivation

There are several different forms of the wave equation, but regardless of its form it is simply a mathematical description of a fixed shape in motion relative to a stationary observer. Eq.1 depicts an arbitrary fixed shape f that is NOT in motion. A stationary observer in an XYZ coordinate system would visualize the amplitude A of the shape, as being transverse (orthogonal) to the X axis, and a function of x.

1. $A = f(x)$ Fixed transverse shape f not in motion

Imagine the observer is positioned at some point x on the X axis, viewing the amplitude A of the fixed shape at the point x. Now imagine an assistant pulling the fixed shape in the positive X direction at a constant velocity c. The observer would see the transverse amplitude A, vary from moment to moment as the fixed shape is pulled past the observer's viewpoint which is fixed at position x. Therefore, the stationary observer's perception of the amplitude A would be as depicted in Eq.2.

2. $A = f(x - ct)$	The most basic form of the wave equation.

Since the function f could apply to any shape, it is not possible to express its time derivative or spatial derivative. However, you can express the relationship between the two derivatives when you consider that in Eq.2 the change in f due to a change in x, is the same as the change in f due to a change in $-ct$. This fact is easily expressed in partial derivative notation as,

3. $\partial f / \partial x = \partial f / \partial(-ct)$ This can be rearranged to give,

4. $\partial f / \partial x = (-1/c)\ \partial f / \partial t$	The first order form of the wave equation.

Applying the above operation again gives us the relationship of the second partial derivatives of f as,

5. $\partial^2 f / \partial x^2 = 1/c^2\ \partial^2 f / \partial t^2$	The second order form of the wave equation.

Voilà !

All three forms of the wave equation.