Richard Alan Satterfield 3/20/18
www.richard-alan.com rev: 5/10/18

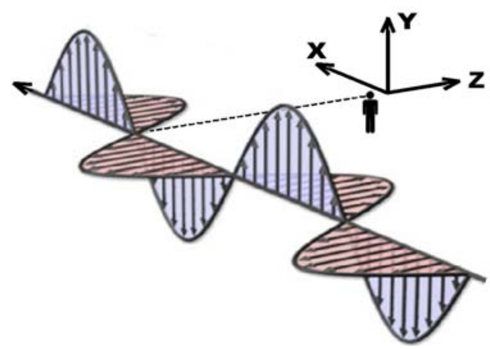
The Wave Equations
For Electromagnetic Waves

Abstract

An often misunderstood aspect of electromagnetic wave propagation is that the field structure of propagating waves is absolutely rigid. This paper addresses that issue head-on by deriving bipolar transverse wave equations based solely on an imaginary model of a rigid shape. A stationary observer is envisioned measuring the amplitude variations of the model's rigid shape as it moves past his viewpoint at a constant linear velocity. The mathematical equations depicting the stationary observers measurements are then compared to Maxwell's equations to show equivalence of equations based on a rigid shape in motion and Maxwell's equations which are based on electromagnetic experiments having nothing whatsoever to do with a shape in motion.

Introduction

There are different forms of the wave equation, but regardless of its form it is simply a mathematical description of a wave shape propagating at constant velocity relative to a stationary observer. A transverse wave is one in which the entity that constitutes the shape of the wave, is oriented orthogonally (transverse) to the direction of propagation. A bipolar transverse wave is a transverse wave in which there are two distinct transverse lobes in the entity that constitutes the shape of the wave. All forms of electromagnetic radiation are thought to be bipolar transverse waves in which one lobe is a magnetic field and the other lobe is an electric field, and in which the two lobes are not only oriented orthogonally to the direction of propagation but also to each other. In addition, the two lobes are thought to have exactly the same shape, that is mirror images of each other. The drawing on the right depicts such a wave graphically. The long black arrow denotes the direction of propagation; it represents a single strand of electromagnetic energy. The arrows perpendicular to the strand represent field strength within the strand and are shaded blue and red to denote the electric field and the magnetic fields respectively. It is important to note that although the wave depicted in the drawing looks to be a sinusoidal shape, it is intended to be representative of any shape.



The utter simplicity of wave propagation like that depicted in the drawing is sometimes lost when the depiction is expressed in mathematically. One reason for this is that wave equations are often derived in a general way so that the wave can have any orientation relative to the orientation of the coordinate system upon which the equations are based. In the above depiction the XYZ coordinate system has been carefully aligned to the orientation of the propagating wave so that the number of terms in the mathematical description will be minimized. Note that the coordinate system orientation shows the wave propagating in the positive X direction, its Y axis aligned to the electric field and its Z axis aligned to the magnetic field. The long black arrow represents a typical strand of energy in a burst of radiation comprised of many strands of electromagnetic radiation, some of which are propagating in a different direction and a different orientation. The actual mix of orientations and directions in a group of strands depends on the source of the radiation. Some sources are designed to emit a specific mix of orientations and directions. In any case each single strand of energy in a burst of radiation, regardless of its orientation and direction, is thought to be a bipolar transverse wave like that depicted above.

Derivation

To assure that the wave equations derived from the above drawing pertain to a propagating wave with a rigid field structure we begin by assuming that the observer in the drawing is actually viewing a stationary (zero velocity) plastic model instead of invisible electric and magnetic fields. The blue and red plastic fins can still be thought of as representing electric and magnetic fields in which the amplitude of the fins (distance the fins protrude from the long black arrow) represents the field strength within the black arrow. Note that the observer's viewpoint shown in the drawing is fixated on a point in the plastic model at $x = 0$, where the fins in the model are both zero amplitude. Now imagine the observer shifting his viewpoint back and forth along the black arrow as he measures the fin amplitudes at each value of *x*. He would obviously conclude that the fin amplitudes were a function of *x*, and that his measurements defined the shape of the fins. His conclusions are expressed mathematically as shown below.

1. $A_y = k_y \; f(x)$	Blue field strength (zero velocity)
2. $A_z = k_z \; f(x)$	Red field strength (zero velocity)

The blue field strength A_y and the red field strength A_z is shown to be proportional to $f(x)$, where $f(x)$ is the shape function based on the observer's fin amplitude measurements. The proportionality constants k_y and k_z, relate the field strength represented by the blue and red fins to the amplitude of the blue and red fins. Note that the field strengths expressed in Eq.1 and Eq.2 are functions of only one variable, *x*. But lets assume the observer has his assistant pull the plastic model in the X direction at a constant velocity *v*. If the observer maintains his viewpoint fixed he would see the amplitudes at his viewpoint change as the variations in the shape of the model were pulled past his fixed viewpoint, and he would conclude that field strength was a function of two independent variables *x* and *vt*, as expressed below.

3. $A_y = k_y \; f(x - vt)$	Blue field strength (velocity *v*)
4. $A_z = k_z \; f(x - vt)$	Red field strength (velocity *v*)

Since the shape function $f(x - vt)$ could represent any shape, it is not possible to express its time or spatial derivatives with any specificity. But you can express the relationship between those two types of derivatives by noting that the change in $f(x - vt)$ due to a change in *x*, is identical to the change in $f(x - vt)$ due to a change in -*vt*. This relationship is easily expressed in partial derivative notation as,

5. $\partial f(x - vt) / \partial x = \partial f(x - vt) / \partial(-vt)$	Derivative with respect to *x* and -*vt*

Since –*v* is a constant we can rearrange Eq.5 to get,

6. $\partial f(x - vt) / \partial x = -1/v \; \partial f(x - vt) / \partial t$	Derivative with respect to *x* and *t*

Next we express the partial derivative with respect to *x* of Eq.3 and Eq.4 to get,

7. $\partial A_y / \partial x = k_y \, \partial f(x - vt) / \partial x$
8. $\partial A_z / \partial x = k_z \, \partial f(x - vt) / \partial x$

In a similar way we express the partial derivative with respect to *t* of Eq.3 and Eq.4 to get,

9. $\partial A_y / \partial t = k_y \, \partial f(x - vt) / \partial t$
10. $\partial A_z / \partial t = k_z \, \partial f(x - vt) / \partial t$

Substituting the right side of Eq.6 for the spatial derivatives on the right side of Eq.7 and Eq.8 we get ,

11. $\partial A_y / \partial x = -k_y / v \;\; \partial f(x - vt) / \partial t$
12. $\partial A_z / \partial x = -k_z / v \;\; \partial f(x - vt) / \partial t$

Substituting from the left side of Eq.9 and Eq.10 to the right side of Eq.11 and Eq.12 we get,

13. $\partial A_y / \partial x = -1/v \;\; \partial A_y / \partial t$
14. $\partial A_z / \partial x = -1/v \;\; \partial A_z / \partial t$

From Eq.3 and Eq.4 we can express the proportionality of A_y and A_z as shown below,

15. $k_z \; A_y = k_y \; A_z$

The proportionality expressed in Eq.15 allows us to rewrite the right side of Eq.13 and Eq.14 to get,

16. $\partial A_y / \partial x = -k_y /(k_z v) \;\; \partial A_z / \partial t$	BIPOLAR TRANSVERSE WAVE EQUATIONS
17. $\partial A_z / \partial x = -k_z /(k_y v) \;\; \partial A_y / \partial t$	(for field strength proportional to fin amplitude)

Maxwell's equations applied to the wave depicted in the drawing (Satterfield[1]) is shown below for comparison.

18. $\partial E_y / \partial x = -\mu_0 \;\; \partial H_z / \partial t$	MAXWELL'S EQUATIONS
19. $\partial H_z / \partial x = -\varepsilon_0 \;\; \partial E_y / \partial t$	(for wave depicted in drawing)

The similarity of the Bipolar Transverse Wave Equations and Maxwell's Equations is evident. The two sets of equations differ only in the constant multiplier of the time derivatives on the right side of the equations. By equating the constant multipliers in the wave equations with the constant multipliers in Maxwell's equations we can derive values for parameters k_y, k_z, and v that would make the two sets of equations absolutely identical. This derivation is expressed in Eq.20 through Eq.25.

20. $k_y /(k_z v) = \mu_0$	multipliers of Eq.16 equated with Eq.18
21. $k_z /(k_y v) = \varepsilon_0$	multipliers of Eq.17 equated with Eq.19

Solving for k_y/k_z in Eq.20 and Eq.21 we get,

22. $k_y/k_z = \mu_0 \, v$
23. $k_y/k_z = 1/(\varepsilon_0 v)$

We now solve for v^2 by equating the right side of Eq.22 with the right side of Eq.23 to get,

24. $v^2 = 1 / (\mu_0 \; \varepsilon_0)$

We now substitute this value for v^2 into either Eq.22 or Eq.23 to get,

25. $(k_y / k_z)^2 = \mu_0 \; \varepsilon_0$

Conclusion

Using the parameter values expressed in Eq.24 and Eq.25 makes the Bipolar Transverse Wave Equations identical to the two Maxwell's Equations that apply to waves in space. This is very significant. The derivation of the wave equations was based solely on geometric considerations of a rigid plastic shape moving at a moderate constant velocity. The derivation of the two corresponding Maxwell's equations was based solely on lab experiments involving electricity and magnetism with no thought whatsoever about wave propagation. And yet, the two sets of equations are identical. Although Maxwell did not have the benefit of our rigid plastic model results, he was aware of the many forms of wave equations pertaining to physical phenomena. You can be sure that when he discovered that two of his equations conformed to one of the wave equations, he would have immediately derived his version of our Eq.24 which related propagation velocity v to the parameters μ_0 and ε_0. He was very familiar with those parameters, and he knew immediately what that relationship signified. His equations had decreed an important commandment.

The propagation velocity of electromagnetic waves shall be exactly the speed of light.

He must have been ecstatic. In his day electromagnetic waves were unknown, and no one had any idea of what light was. Maxwell conjectured that light might actually be an electromagnetic wave. But I wonder if Maxwell realized that his equations had decreed a second commandment.

The field structure of electromagnetic waves shall be absolutely rigid.

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References

1. *The Birth of a Wave. Satterfield, www.richard-alan.com* Section 3 derives Maxwell's equations applied to a wave in which the XYZ coordinate has been carefully aligned to the polarization of the wave.